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Tensegrities and rotating rings of tetrahedra: a symmetry viewpoint of structural mechanics

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Symmetry is a common attribute of both natural and engineering structures. Despite this, the application of symmetry arguments to some of the basic concepts of structural mechanics is still a novelty. This paper shows some of the insights into structural mechanics that can be obtained through careful symmetry arguments, and will show how these can provide a key to understanding the paradoxical behaviour of some symmetric structures.

Keywords: symmetry; structural mechanics; mechanisms

1. Introduction

It is commonly understood that the key to structural rigidity is to build structures that are triangulated, and structural engineers will understand this to be true for the most basic structural simplification, that a structure is pin-jointed. In 1864, James Clerk Maxwell published this idea as a simple rule that described the number of bars a pin-jointed structure requires for rigidity. However, as Maxwell was aware, there are a number of structures for which this simple Maxwell rule does not work (Maxwell 1864). In particular, some symmetric structures apparently have too few bars, and yet are rigid, while other symmetric structures that apparently have enough bars are not rigid. In retrospect, this is understandable: group theory, the basic mathematical language used to understand symmetry, was not developed for another 30 years.

The two structures to which the title refers both have paradoxical properties. Tensegrities, such as the structure shown in figure 1, are rigid structures that often appear to have too few constraints to enforce this rigidity. The ring of six tetrahedra shown in figure 2, by contrast, appears to have enough constraints to ensure that it is rigid, and yet, in fact, it is a finite mechanism, and can keep rotating through itself indefinitely. In both of these cases, the symmetry of the structure proves to be a key to understanding its paradoxical behaviour.

This paper is split into five sections. Section 2 describes some of the relevant background in structural mechanics and symmetry, and also describes Maxwell's rule and how it can be defined in a symmetry-extended form. Sections 3 and 4 then describe the applications of this theory to a number of paradoxical structures: the idealized structures to which the title refers; the timber octagon at Ely Cathedral, and the crystal structure of quartz.

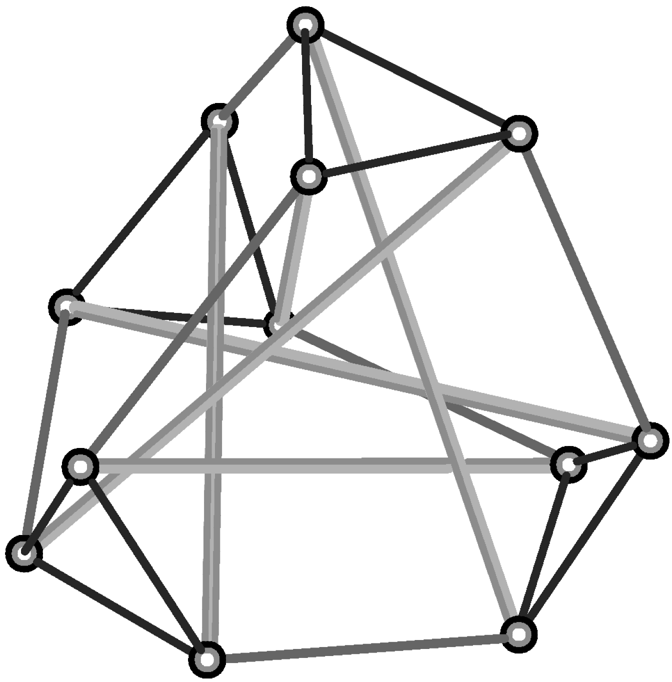

Figure 1. An example of an underconstrained tensegrity structure. The outer net of members are cables in tension. The inner bars are struts in compression.

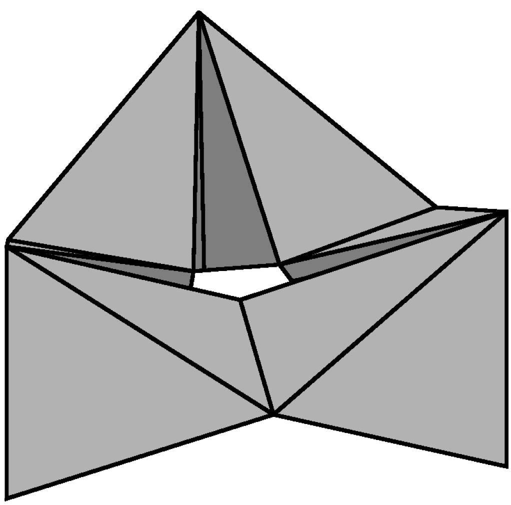

Figure 2. A ring of six tetrahedra.

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2. Background

(a) *An introduction to structural mechanics*

For simplicity, this paper will almost always make the simplest of structural assumptions, that the structures considered are *pin-jointed*, and are only loaded at their

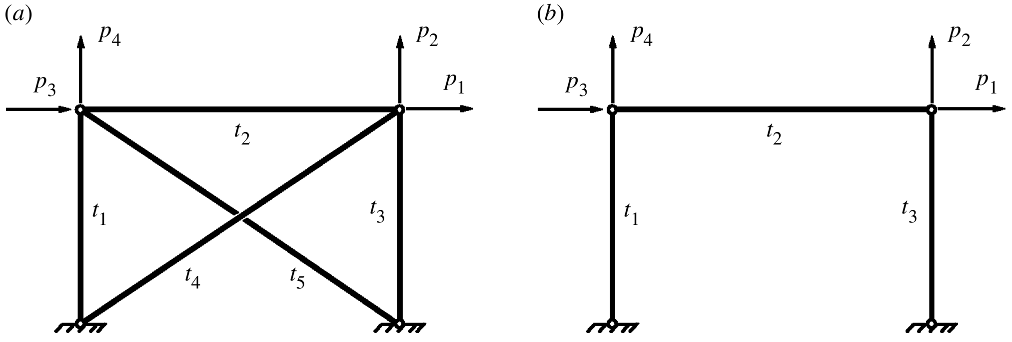

Figure 3. Two pin-jointed structures. (a) Statically indeterminate structure with one state of self-stress (note that the two diagonal bars do not meet). (b) Kinematically indeterminate structure that allows one mechanism. p_1–p_4 are used to represent external loads; if they were replaced by d_1–d_4 the same coordinate system could also be used for joint displacements. t_1–t_5 are used to represent internal bar tensions; if they were replaced by e_1–e_5 the same coordinate system could also be used for bar extensions.

joints. In this case, the *internal* state of the structure can be fully described by the tension in each of the bars, and these tensions must be in *equilibrium* with the *external* forces applied at the joints. It is usually assumed that these are linear relationships, and, hence, that there is an equilibrium matrix $\mathbf{H}$ describing the relationship between vectors of the internal tensions $\mathbf{t}$ and the externally applied forces $\mathbf{p}$: $\mathbf{H}\,\mathbf{t} = \mathbf{p}$.

Structures are referred to as *statically determinate* if there is a unique solution to the equilibrium equations for any applied loading. If a structure is statically *indeterminate*, then the structure will admit different *states of self-stress*, where the structure can be stressed against itself, with no external load applied.

Figure 3a shows a simple example of a statically indeterminate structure. If the two diagonal members were shortened, e.g. by turnbuckles, they would go into tension, and the outer bars into compression, even if no external loads were applied at the joints.

As well as equilibrium relationships, structural mechanics is also concerned with the geometry of any small deformation of the structure, known as the *compatibility* between the extensions of the bars and the displacement of the joints. Again this is usually assumed to be a linear relationship, with a compatibility matrix $\mathbf{C}$ describing the relationship between the displacements of the joints $\mathbf{d}$, and the extension of the members $\mathbf{e}$: $\mathbf{C}\,\mathbf{d} = \mathbf{e}$. A simple proof based on energy methods shows that $\mathbf{C} = \mathbf{H}^{\mathrm{T}}$; there is a very close relationship between the statics (equilibrium) and kinematics (deformations) of a structure.

Structures are referred to as *kinematically determinate* if there is a unique solution to the compatibility equations for any set of internal extensions. If a structure is kinematically *indeterminate*, then there will be certain movements of the joints where, at least to the first-order approximation made, there are no changes in bar lengths. This is referred to as a *mechanism*. If the structure eventually tightens up as the mechanism is displaced, i.e. the mechanism involves higher than first-order changes in bar length, the mechanism is called infinitesimal. Otherwise the mechanism is called finite.

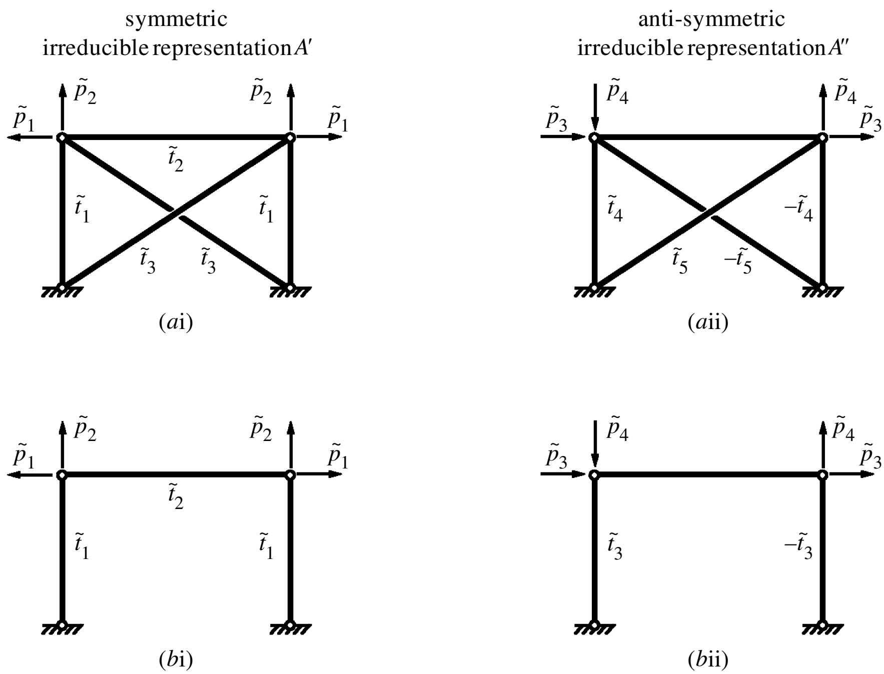

Figure 4. Symmetry-adapted coordinate systems, denoted by a tilde.

Figure 3*b* is a simple example structure that allows a finite motion. In mechanical engineering terms, it is a planar four-bar linkage.

(*b*) *An introduction to symmetry*

A *symmetric structure* is one that is left unaltered by a symmetry operation. These operations may be reflections, rotations, translations or dilations, together with the identity, which simply leaves everything untouched. All of the possible symmetry operations for a structure together make a symmetry *group*. Although there is an infinite variety of structures, the number of symmetry groups is strictly limited. One great advantage of using symmetry in analysis is that, for each of the symmetry groups, most of the hard work has already been completed by mathematicians, and has been collected together in reference works such as Altmann & Herzig (1994).

The key to applying symmetry to structural mechanics is understanding how symmetry can be used to change the coordinate systems used for representing, for example, applied loads or tensions, into a symmetry-adapted form. This is well understood for the sort of simple bilateral symmetry of the structures shown in figure 3. Both external, and internal, coordinate systems can be split into what are commonly termed *symmetric* components that are unchanged by the reflection in the central plane of symmetry, and *antisymmetric* components that are reversed by the symmetry operations. Symmetry-adapted coordinate systems for the example structures in figure 3 are shown in figure 4.

Mathematicians will recognize that the results shown in figure 4 are the result of splitting the *reducible representations* shown in figure 3 into *irreducible* representations for the particular symmetry group of these structures. These structures are

Table 1. *Irreducible representations of* C_s

	identity	reflection
A' (symmetry)	1	1
A'' (antisymmetry)	1	−1

unchanged by the identity and reflection in a central plane; these operations together are described as the symmetry group C_s. The irreducible representations of C_s are shown in table 1, which will be found in more abstract form in, for example, Altmann & Herzig (1994). This table essentially describes the familiar idea of 'symmetry and antisymmetry'. However, while it is not intuitive to see how symmetry and antisymmetry can be extended to more complicated symmetry groups, there are tables corresponding to table 1 for *all* symmetry groups. Using these tables, calculating symmetry-adapted coordinate systems that reflect different aspects of the symmetry becomes straightforward for all symmetry groups (Kangwai *et al*. 1999; Kangwai & Guest 2000).

(*c*) *Maxwell's rule*

In 1864 James Clerk Maxwell published an algebraic rule setting out a condition for a pin-jointed frame composed of b rigid bars and j joints to be both statically and kinematically determinate (Maxwell 1864). The number of bars needed to stiffen a three-dimensional frame free to translate and rotate in space as a rigid body is

$$b = 3j - 6. \tag{2.1 a}$$

The physical reasoning behind the rule is clear: each added bar links two joints and removes at most one internal degree of freedom. The rule simply equates the number of external and internal degrees of freedom, shown in, for example, figure 3. It is trivial to modify Maxwell's rule for other simple cases: for a three-dimensional frame fixed to supports,

$$b = 3j; \tag{2.1 b}$$

for a two-dimensional frame free to translate and rotate in plane,

$$b = 2j - 3; \tag{2.1 c}$$

for a frame fixed to supports and confined to a plane,

$$b = 2j. \tag{2.1 d}$$

All four of these cases are covered by a formulation such as equation (2.2) with appropriate values of t and $\varDelta$:

$$b = tj - \varDelta. \tag{2.2}$$

As Maxwell himself noted, equation (2.2) is a necessary but not, in general, a sufficient condition for establishing determinacy. A full account of the degrees of freedom of the frame must allow for the possibility of states of self-stress and mechanisms,

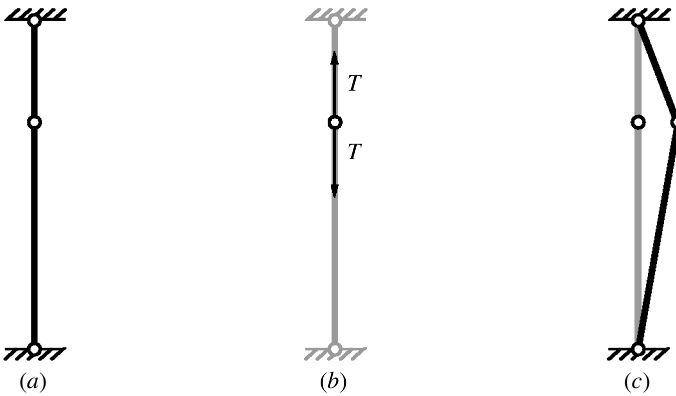

Figure 5. (*a*) A simple structure that satisfies Maxwell's rule, and yet is statically and kinematically indeterminate; (*b*) the state of self-stress; (*c*) the infinitesimal mechanism.

and so the full inventory of degrees of freedom of the frame can be written as an extended Maxwell's rule (Calladine 1978):

$$b - tj + \Delta = s - m, \tag{2.3}$$

where s and m count the states of self-stress and mechanisms, respectively, and can be determined by finding the rank of the equilibrium matrix that describes the frame in a full structural analysis (Pellegrino & Calladine 1986).

Maxwell's rule, or the extended rule, can easily be applied to the structures shown in figure 3; equation (2.1 *d*) is the appropriate form. For figure 3*a*, the structure has $j = 2$ and $b = 5$. Thus there is one more bar than required, and there is a single state of self-stress. Figure 3*b* has the same number of joints, $j = 2$, but two less bars, $b = 3$. There is now one less bar than required, and, hence, a single mechanism.

Figure 5 shows a simple example of a structure that satisfies Maxwell's rule, with $j = 1$ and $b = 2$, but is, in fact, both statically and kinematically indeterminate; an examination of the equilibrium or compatibility matrix would show the matrix to be rank deficient. There is a state of self-stress where both bars are in tension, and also an infinitesimal mechanism, where, to a first-order approximation, the bars do not change length as the central joint moves sideways. In fact, infinitesimal mechanisms of this type can be classified into different categories (Connelly & Whiteley 1996), and this example is known as pre-stress stable. If the tension in the bars is $T > 0$, then some first-order stiffness is imposed on the mechanism. As we shall see later, underconstrained tensegrity structures are also pre-stress stable, but, in these cases, a single highly symmetric state of self-stress can stiffen many different infinitesimal mechanisms.

(*d*) *A symmetry extension of Maxwell's rule*

Fowler & Guest (2000) will shortly publish a symmetry extension of Maxwell's rule. It goes beyond the original rule by not only requiring that the number of internal and external degrees of freedom are numerically equal, but also that they are equisymmetric. In its simplest form, the rule, written in the language of representations, is

$$\Gamma(b) - \Gamma(e) = \Gamma(s) - \Gamma(m), \tag{2.4}$$

where $\Gamma(b)$ is the representation of the symmetry of the bars, and $\Gamma(e)$ is the representation of the symmetry of the external coordinate system; Fowler & Guest (2000) show that these representations can easily be found for any given structure by counting the bars and joints that remain unmoved by symmetry operations applied to the structure. $\Gamma(s)$ and $\Gamma(m)$ are the representations of the states of self-stress, and the mechanisms, respectively.

For the example structures in figure 3, and examining the symmetry-adapted internal and external coordinate systems shown in figure 4, the representations of the bars and external coordinate systems can be written down. For both structures, the external coordinate system has representation $\Gamma(e) = 2A' + 2A''$: two of the coordinates of the symmetry-adapted coordinate system are symmetric, two are antisymmetric.

For figure 4*a*, $\Gamma(b) = 3A' + 2A''$: three of the coordinates of the internal symmetry-adapted coordinate system are symmetric, and two are antisymmetric. Applying the symmetry extension of Maxwell's rule, this gives $\Gamma(s) = A'$, showing that the state of self-stress must be symmetric. Similarly, for figure 4*b*, $\Gamma(b) = 2A' + A''$: two of the coordinates of the symmetry-adapted internal coordinate system are symmetric, and one is antisymmetric. Applying the symmetry extension of Maxwell's rule, this gives $\Gamma(m) = A''$, showing that the mechanism must be antisymmetric.

For these structures, the symmetry-adapted Maxwell's rule simply gives more information about the symmetry of the state of self-stress or mechanism that has already been detected by the original rule. Applying the symmetry-adapted rule to figure 5, however, allows the detection of the mechanism and state of self-stress. This structure again has symmetry C_s; it is left unchanged by the identity and reflection in the vertical plane that passes along the bars. For this structure, $\Gamma(b) = 2A'$ and $\Gamma(e) = A' + A''$, and, hence, $\Gamma(s) - \Gamma(m) = \Gamma(b) - \Gamma(e) = A' - A''$. Thus $\Gamma(s) = A'$, the structure has a symmetric state of self-stress (as shown in figure 5*b*), and $\Gamma(m) = A''$, the structure has an antisymmetric mechanism (as shown in figure 5*c*).

An interesting point about the symmetry analysis of figure 5 is that it is only valid for the particular symmetric configuration shown. If the structure is displaced in the direction of the infinitesimal mechanism, the symmetry is broken, and a Maxwell analysis will no longer predict a mechanism. In fact, the mechanism no longer exists: it was an infinitesimal mechanism that only existed for a first-order analysis in the initial symmetric configuration.

3. Overconstrained mechanisms

There are a number of structures that satisfy Maxwell's rule, and yet are kinematically indeterminate, and allow mechanisms; figure 5 was a simple example. In this case, the infinitesimal motion allowed by the structure is physically intuitive; what is surprising, however, is that some structures that satisfy Maxwell's rule in fact allow *finite* motion. Symmetry can be the key to this motion (Kangwai & Guest 1999), and three different examples will be examined.

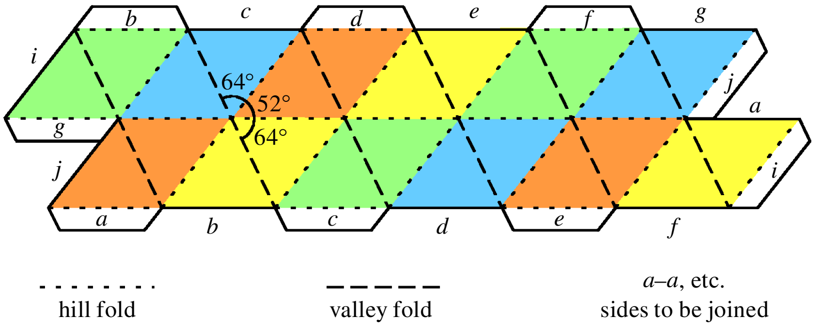

Figure 6. How to make a ring of six rotating tetrahedra. The figure is colour coded to correspond to figure 7. The particular angles shown just allow the ring to pass through itself, as shown in figure 7; figure 2, by contrast, has tetrahedra made of isosceles triangles where the smallest angle is less that 52°.

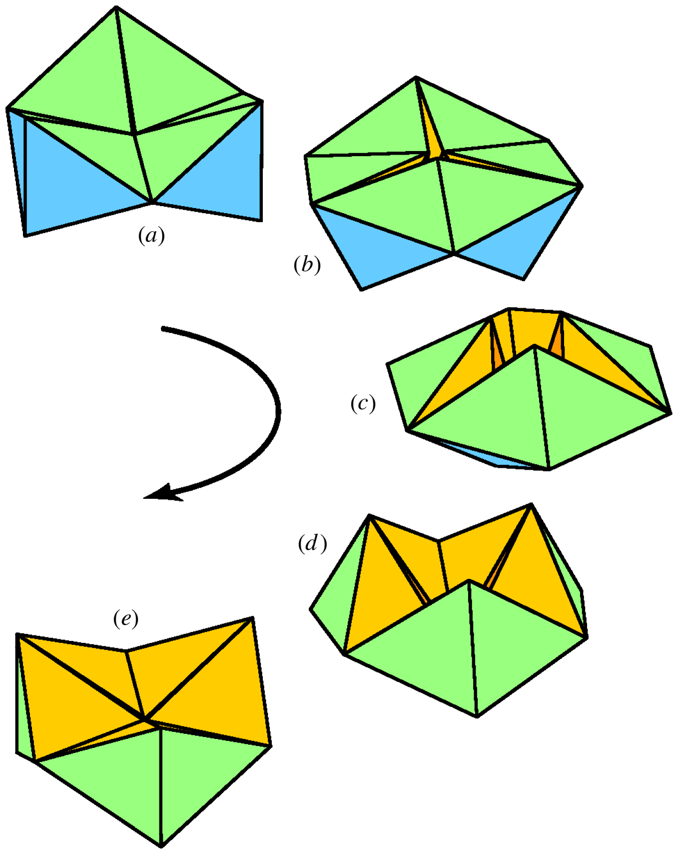

Figure 7. Finite motion of the ring of six rotating tetrahedra, showing one quarter of a complete cycle: (*a*) D_{3h} high-symmetry point; (*b*) generic C_{3v} symmetry; (*c*) D_{3d} high-symmetry point; (*d*) generic C_{3v} symmetry; (*e*) D_{3h} high-symmetry point.

Figure 8. The timber octagon at Ely Cathedral. © Crown copyright. NMR.

(a) Rings of rotating tetrahedra

Rings of rotating tetrahedra are well-known structures that show a surprising continuous motion, where the tetrahedra continuously rotate through the centre of the ring. It is easy to build a ring of tetrahedra from a single sheet of material, and instructions for the example with six tetrahedra are given in figure 6. Rings can be made with any even number of tetrahedra greater that six, but we shall concentrate on the case with six tetrahedra, because this is an example that, despite its continuous motion, satisfies Maxwell's rule for determinacy. Consider the structure shown in figure 2 as made up of pin-jointed bars along the edges of the tetrahedra. This leaves each tetrahedron rigid, but allows neighbouring tetrahedra to rotate relative to one another about the common edge. Counting the bars and joints gives $b = 30$, $j = 12$. The relevant form of Maxwell's rule, $b = 3j - 6$, is clearly satisfied.

The finite motion of the six tetrahedra is shown in figure 7. Excluding rigid-body motions, the path followed is unique and any other motion would require deformation of the tetrahedra. Generically, as shown in figure 7b, d, the structure has $C_{3\text{v}}$ symmetry: it is left unchanged by the identity, rotation by 120° or 240°, or reflection in three vertical planes of symmetry. The structure does, however, pass through two types of high-symmetry point. In figure 7a, e, the structure has an additional horizontal plane of symmetry; this symmetry group is known as $D_{3\text{h}}$. In figure 7c, the structure has three additional twofold rotation axes in the horizontal plane; this symmetry group is known as $D_{3\text{d}}$.

An analysis using the symmetry-extended form of Maxwell's rule reveals the existence of a mechanism. Consider, initially, the ring in the position shown in figure 7a,

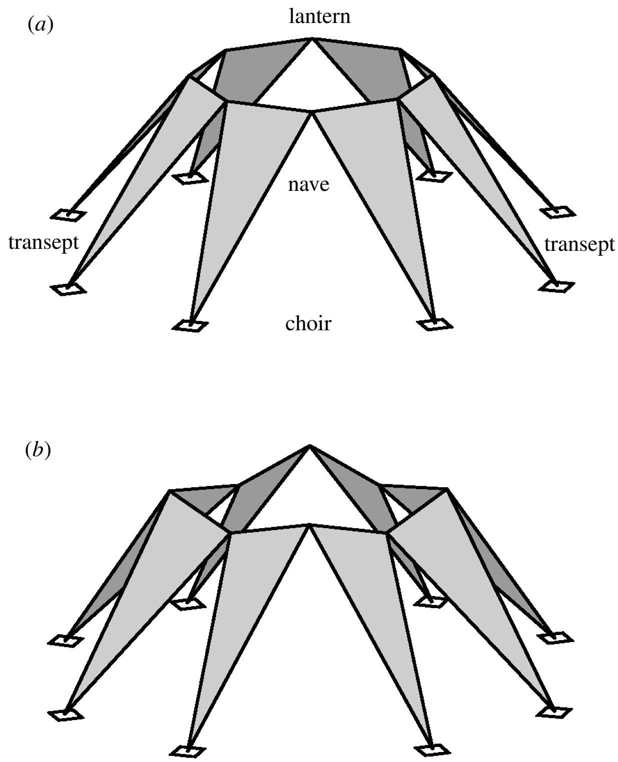

Figure 9. (a) A simplified view of the octagon. The main structural timbers run along the edge of the triangles shown. (b) The structure displaced a small distance in the direction of its mechanism.

where it has $D_{3\mathrm{h}}$ symmetry. The table of irreducible representations for this symmetry group shows that it is possible to split the external and internal degrees of freedom into six groups, each representing different aspects of the full symmetry of the structure. Doing so reveals a mechanism with $C_{3\mathrm{v}}$ symmetry, while the state of self-stress has distinct, D_3 symmetry: it is preserved by the threefold rotations about the vertical axis, the two-fold rotation about three axes in the horizontal plane, but not reflection in the vertical planes.

At this point, this example appears little different from the structure in figure 5; the existence of a mechanism has been discovered, but mobilizing it destroys the symmetry used and, hence, the validity of the analysis. In this case, however, symmetry is not entirely destroyed, and it is possible to re-analyse the structure using the 'lesser' $C_{3\mathrm{v}}$ symmetry of the mechanism. Doing this shows that the mechanism does indeed have $C_{3\mathrm{v}}$ symmetry, and this must be true not only at the initial point, but also along the finite path shown in figure 7. The state of self-stress now only has C_3 symmetry (threefold rotation, but no reflection) at most points along the path—it is essentially a twist—and this self-stress cannot prevent the finite motion. Thus, although the structure passes through points of higher symmetry along the path, it is the analysis using the essential $C_{3\mathrm{v}}$ symmetry of the mechanism that is the key to understanding the paradoxical behaviour.

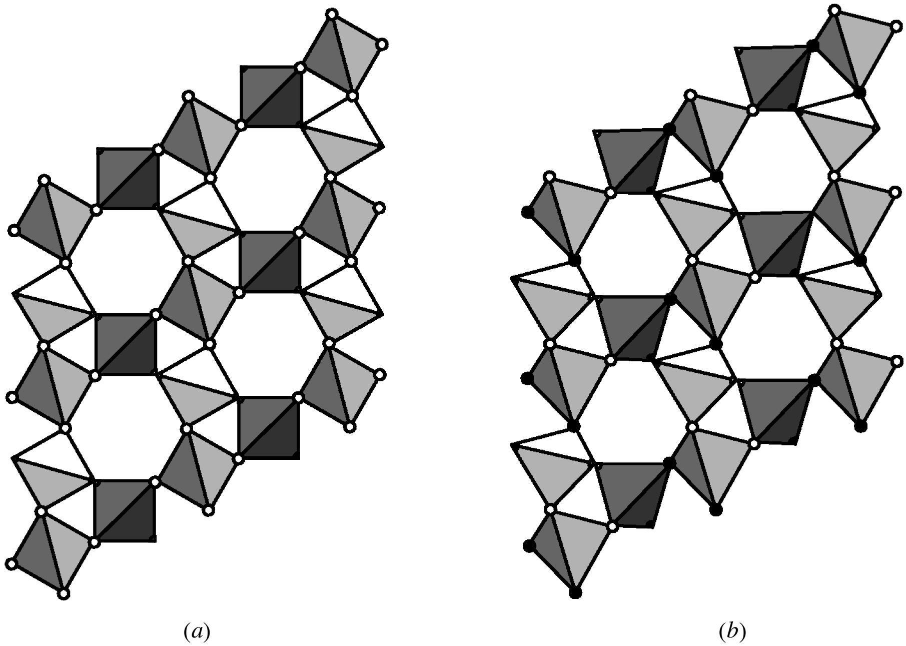

Figure 10. The crystal structure of quartz shown as a framework of SiO_4 tetrahedra. (*a*) The high-temperature (β) phase. (*b*) The low-temperature (α) phase. The tetrahedra shown lie in three planes parallel to the paper, the darkest tetrahedra in the lowest plane. The tetrahedra are linked in spirals; complete circles at nodes denote tetrahedra that are linked, partial circles are where tetrahedra are linked to planes above or below those shown. (*b*) White nodes have moved up and black nodes down in the transition from (*a*).

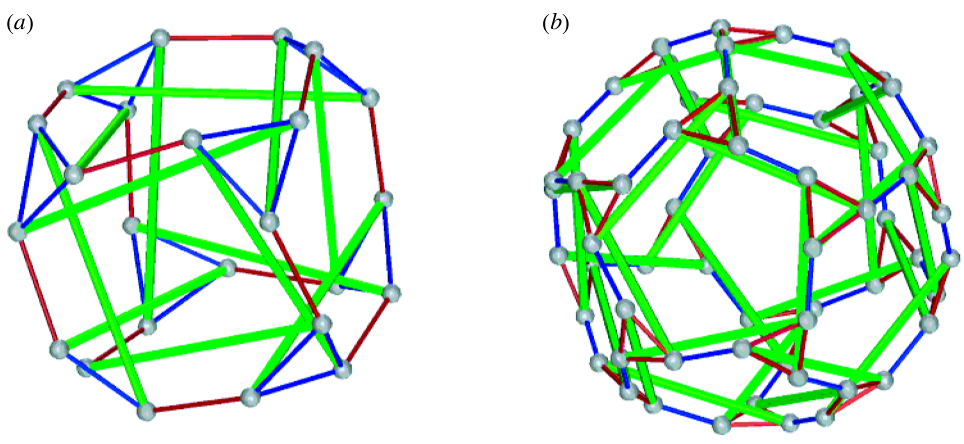

Figure 11. Underconstrained tensegrities from Connelly & Back's (1998) catalogue: (*a*) has the rotational symmetries of a cube and (*b*) has the rotational symmetries of an icosahedron. The green members are struts in compression, the red and blue are cables in tension. Every member of each colour can be superposed on every other by one of the operations of the symmetry group.

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(b) *The timber octagon of Ely Cathedral*

An interesting example of an overconstrained mechanism is the timber octagon at Ely Cathedral, which is shown in figure 8. The octagon stands above the crossing of the cathedral and was built to replace a collapsed tower. The main structure of the octagon was built around 1334, and has thus stood for two-thirds of the millennium; and is twice as old as this journal, the *Philosophical Transactions of the Royal Society*! Despite this, Wade & Heyman (1985), in a careful examination of the structure, described it as 'an architectural masterpiece, but it was, in its original form, something of a structural mistake'.

A simplified view of the main structure of the octagon is shown in figure 9*a*. A count of the bars and joints gives $b = 24$ and $j = 8$, and the structure satisfies the relevant form of Maxwell's rule (equation (2.1 *b*)). However, the structure actually allows the finite mechanism shown in figure 9*b* (Tarnai 1988). Re-analysing the structure using the symmetry extension of Maxwell's rule shows that this is indeed the case. There is a finite mechanism with the full $C_{4\mathrm{v}}$ symmetry of the structure (a fourfold axis of rotation, and four vertical planes of reflection), while the state of self-stress has lesser C_4 symmetry, without the planes of reflection.

The existence of this mechanism raises the question of how the structure has stood for so long, as it could not support any load with a component parallel to the mechanism. It is probable that the octagon initially adjusted its shape to ensure that the weight distribution would not excite the mechanism. Since then, frequent repairs, and the addition of large amounts of bracing, have ensured that it will survive into the next millennium.

(c) *Phase transitions in quartz*

A final example of an overconstrained mechanism can be found in the crystal structure of quartz. Quartz can be considered to be composed of rigid SiO_4 tetrahedra that are linked by corner-sharing oxygen atoms in an infinite framework. Two phases of quartz are shown using this model in figure 10 (Giddy *et al*. 1993).

A simple Maxwell count of the internal and external degrees of freedom of this structure shows that it should be statically and kinematically determinate. However, again because of the symmetry, this is not the case and a mechanism allows the transition between the two phases shown in figure 10 (Dove 1997).

4. Tensegrity structures

Underconstrained tensegrity structures are, in some ways, the opposite of the overconstrained mechanisms described in the previous structure. According to Maxwell's rule, these structures do not have enough members for them to be rigid. However, using the terminology of Connelly & Whiteley (1996), they are 'prestress stable', and can be stiffened by a state of self-stress.

Tensegrity structures were first invented by Kenneth Snelson, and were popularized by Buckminster Fuller (Marks 1960). 'Tensegrity' can be a rather vague term, but in this paper it will be used to refer to underconstrained structures that can be made rigid in a particular symmetric configuration by a state of self-stress. Examples of these tensegrity structures are shown in figures 1 and 11. One of the reasons for the popularity of tensegrity structures is that when prestressed, any of the members that are in tension may be replaced by a cable, which often allows the compression members of the structure to be entirely separated from one another.

In general, tensegrity structures have many infinitesimal mechanisms. A Maxwell bar and joint count shows the structure in figure 1 to have at least six independent mechanisms, the structure in figure 11*a* to have at least 18 independent mechanisms, and the structure in figure 11*b* to have at least 54 independent mechanisms. In fact, a full symmetry-based structural analysis shows that they each have a state of self-stress with the full symmetry of the structure, and, hence, equation (2.3) implies that each must have an additional mechanism beyond these numbers.

Comparing the tensegrity structures with the finite mechanisms in § 3, the key distinction is that the tensegrity structures have a state of self-stress with the full symmetry of the structure. The finite mechanisms could be mobilized because the state of self-stress had some lesser symmetry than the mechanism. With tensegrity structures, all the mechanisms must have the same or less symmetry than the state of self-stress. All of the mechanisms are stiffened by the state of self-stress.

Connelly & Back (1998) have studied a mathematical idealization of tensegrity structures that are made up of idealized struts and cables. The cables are members that only provide tension, and have a rest length of zero, while the struts are members that can only be in compression, and have an infinite rest length. Using the initial assumption that there is a state of self-stress with particular symmetry properties, and assuming that, for instance, there is only one type of strut, but two different types of cable, Connelly & Back (1998) have produced a catalogue showing the different tensegrities that are possible for each symmetry group. Figure 11 shows two of the colour-coded structures from Connelly & Back's (1998) catalogue; figure 1 was also taken from the catalogue.

5. Conclusions

One of the beauties of structural mechanics is that it simplifies seemingly intractable problems to a stage where, for instance, engineers can sensibly and safely design buildings. By putting structural mechanics in a group-theoretic context, this simplification can be extended further for many complicated structural systems.

As well as providing the insights shown in this paper, it may be hoped that the methods described will prove to be of further use in simplifying difficult problems in, for instance, complicated crystal or protein structures.

I thank R. D. Kangwai, S. Pellegrino, P. W. Fowler and R. Connelly for many helpful discussions on symmetry and its application to structural analysis. I would also like to thank R. Connelly and R. Back for preparing special versions of the tensegrity figures from their catalogue.

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Simon Guest is 31. He was born in Wolverhampton, and studied Engineering at Cambridge, obtaining a BA with distinction in 1990. His initial research was also at Cambridge, developing novel deployable structures. He was awarded a prize by IUTAM for the best paper in solid mechanics by a younger scientist at their 1992 Congress, and he completed his PhD in 1994. In 1993, Simon was appointed as a University Assistant Lecturer at Cambridge. In 1996, he spent six months at Stanford University's Department of Mechanics and Computation. He was promoted to University Lecturer in 1997, and was awarded an Esso Engineering Teaching Fellowship in 1999. His research interests include the development of deployable structures, and novel techniques for the computational analysis of structures.